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Book Review

Ding-Zhu Du and Panos M. Pardalos (eds.), *Minimax and Applications*, Nonconvex Optimization and Its Applications Vol. 5, Kluwer Academic Publishers, Dordrecht, 1995, 292 pages.

It has long been recognized that classical minimax theory due to Von Neumann, together with duality and saddle point analysis, has played a crucial role in the development of the fields of optimization and game theory. Today researchers and practitioners are recognising that minimax theory, methods and applications appear in a broad spectrum of additional disciplines, including pure mathematics, computational complexity, networks, location theory, optimal control, scheduling and information theory, to name a few.

The papers collected in this volume serve to inform the reader of both the classical and more recent developments in the areas of minimax and its applications. Readers of this book will be impressed with the great diversity of pure and applied disciplines to which minimax theory makes important contributions.

Each of the eighteen papers in this book is written by one or more of the leading researchers in minimax theory and its applications. These papers can be roughly classified into three categories. In particular, each paper gives either a survey, new analytical or algorithmic results, or new applications.

Three of the papers give valuable surveys of use to both novices and experts in the minimax field. The first of these survey papers, by S. Simons, traces the history, development, and proofs of the most important minimax theorems, starting with the original result of Von Neumann of 1928. A second survey by C.G. Diderich and M. Gengler provides an overview of theoretical results on minimax game trees and of algorithms that have been proposed for solving them. The third survey by F. Cao *et al*. gives brief summaries of a number of combinatorial problems that can be formulated as minimax problems, together with references and open questions regarding these problems.

There are seven articles that give either new algorithmic or analytical results concerning the solution of minimax problems. L. Qi and W. Sun present a new iterative method for minimax problems that solves a sequence of linear-quadratic programming problems in order to find a saddle point of the original minimax problem. Taking a somewhat different approach, J.F. Sturm and S. Zhang propose an algorithm for convex minimax problems that is based on solving a dual formulation of the problem by an interior point method. The articles by F. Cao and by B. Chen and G. J. Woeginger concern special algorithms for some minimax problems for minimizing makespans that arise in production scheduling. In the former paper, Cao shows a new worst-case performance bound for the heuristic for minimizing makespan developed in 1978 by Coffman, Garey and Johnson. In the latter paper, the authors propose and analyze heuristic algorithms for minimizing makespans in certain open shops and in certain flow and job shops. K.-I. Ko and C.-L. Lin show that the computational complexity of a variety of minimax problems is characterized by the second level of the polynomial-time hierarchy. X.D. Hu and F.K. Hwang give a competitive algorithm for a difficult version of the special minimax problem known as the counterfeit coin problem, and L. N. Vicente and P. H. Calamai derive necessary and sufficient conditions for minimax programs by way of a bilevel programming formulation.

Eight of the papers give new applications of minimax theory. D.F. Hsu *et al.* show how to view the Steiner ratio problem with rectilinear distance measure as a maximin problem, and they derive new lower and upper bounds for this problem. S.-H. Teng proposes minimax models and approximate solution methods for a type of sampling problem called mutually repellant sampling. In a different vein, T. Helgason *et al.* construct two new maximin optimization models for carrying out the allocation of seats in national parliaments by the fairest means possible. G. Xue and S. Sun show how to solve a special maximin application in Euclidean three-space called the spherical one-center problem. In two companion articles, A.W.M. Dress *et al.* and L. Yang and Z. Zeng compute for the first time certain numbers called Heilbronn numbers whose values are given by solving difficult geometric minimax problems. The article by G. Yu and P. Kouvelis analyzes the computational complexity of three types of applied discrete minimax problems and shows that, under certain assumptions, pseudo-polynomial algorithms can be found for these problems. Finally, J. Gu gives a new minimax relaxation search algorithm for constrained global optimization that has certain advantages over local search and simulated annealing methods.

This is a valuable book carefully written in a clear and concise fashion. The survey papers give coherent and inspiring accounts of some of the key minimax theory and applications, and the coverage of algorithmic and applied topics provided by the remaining articles is impressive. Important and penetrating advances in the minimax field are given in this book. Both graduate students and researchers in fields such as optimization, computer science, production mangement, operations research and related areas will find this book to be an excellent source for learning about both classic and more recent developments in minimax and its applications. The editors are to be commended for their work in gathering these papers together. I strongly recommend this book to anyone interested in minimax theory and its applications.

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